

A Linear-Nonlinear Sliding Mode Control of Finite Time Trajectory Tracking for Multirotor UAVs Using the Logarithmic Map of SO(3)

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Abstract: This work presents a control scheme for trajectory tracking of multirotor UAVs featuring finite-time convergence of both position and attitude. The attitude error is expressed in the Lie algebra via the logarithmic map of SO(3), which transforms geodesics on the rotation manifold into straight lines in the Lie algebra, thereby providing the most natural and effective representation of attitude error. Both the position and the attitude controllers are designed based on a unified framework of linear-nonlinear sliding-mode control where the coexistence of the linear and fractional-exponent terms induces an adaptive two-phase dynamic behavior: a smooth decay is governed by the linear component when the error is relatively large, while the nonlinear fractional term accelerates convergence as the state approaches the neighborhood of the origin. In both regions, the corresponding dominant term ensures that the tracking error continues to converge rapidly toward zero. Computer simulations were conducted to validate the approach, and the preliminary results demonstrated the effectiveness of the proposed method supporting its feasibility in the UAV applications.

Keywords: Multirotor UAV, Linear-Nonlinear Sliding Mode Control, Finite-Time Stability, Logarithmic Map Of SO(3)

1. INTRODUCTION

Multirotor unmanned aerial vehicles (UAVs) have experienced rapid growth in both academic research and real-world deployment, owing to their high maneuverability and vertical takeoff capability (Kyrkou et al., 2019). With the increasing maturity of open-source autopilot and simulation frameworks (e.g., PX4 and ROS2), the implementation of advanced control algorithms has become highly feasible. This leads to a growing demand for high-performance flight controllers capable of aggressive maneuvers and precise trajectory tracking, where robust and accurate attitude control remains a central challenge in multirotor research.

Various attitude representations have been used to describe rigid-body rotations, including Euler angles, Rodrigues parameters, quaternions, and the special orthogonal group SO(3). Euler angles suffer from gimbal-lock singularities, while quaternions avoid singularities but introduce ambiguity due to their double-cover nature. In contrast, geometric control on SO(3) offers a globally valid and coordinate-free framework, providing a consistent foundation for error representation and stability analysis on the Lie group (Chaturvedi et al., 2011). This has

established geometric control as a compelling approach for multirotor attitude regulation.

Within this research direction, logarithmic-map-based error representations have been shown to provide notable advantages. Johnson and Beard (Johnson and Beard, 2022) demonstrated that the logarithmic map transforms geodesics on SO(3) into straight lines in the Lie algebra, making it a natural and effective way to define attitude errors for control purposes. Building on this concept, Chuang (Chuang, 2025) introduced an SO(3)-based geometric controller that combines logarithmic attitude errors with integral terminal sliding surfaces. However, its closed-loop dynamics only achieved asymptotic stability, and the convergence rate remained limited near equilibrium.

Separately, sliding-mode control (SMC) and its variants have attracted considerable interest due to their robustness and inherent finite-time convergence properties. Techniques such as nonsingular terminal SMC, fast nonsingular terminal SMC, integral SMC, and super-twisting SMC have been applied to enhance disturbance rejection, improve tracking performance, and increase fault tolerance (Nguyen et al., 2021; Wang et al., 2019; Qiao and Zhang, 2020; Karahan et al., 2023; Rivera et al., 2011).

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However, most of the existing methods are still formulated in Euclidean coordinates (Hashim, 2019). Although this formulation is convenient, it does not reflect the intrinsic geometry of rotational motion. In contrast, the rotation manifold $\text{SO}(3)$ provides a coordinate-free and geometrically consistent space for attitude representation. The logarithmic map further transforms geodesic curves on $\text{SO}(3)$ into straight lines in its tangent space, offering a natural foundation for geometric error dynamics and stability analysis (Johnson and Beard, 2022). This motivates the adoption of a geometric control framework that operates directly on the manifold rather than in local coordinates.

These observations highlight a key research gap: while geometric control on $\text{SO}(3)$ offers a globally consistent representation and sliding-mode-based approaches provide finite-time convergence, their integration within a unified framework remains insufficiently investigated. In particular, the geometric structure induced by the logarithmic map offers a linear-like representation of the attitude error in the Lie algebra, making it highly suitable for the development of finite-time control laws.

Motivated by the insight, this work presents a unified finite-time multirotor control framework in which both the translational and rotational subsystems are designed based on the linear-nonlinear sliding-mode structure originally introduced by Yogi (Yogi et al., 2021). Unlike conventional Euclidean formulations, the proposed method integrates this finite-time structure with the logarithmic-map representation on $\text{SO}(3)$, enabling geometric consistency while achieving rapid convergence near equilibrium.

The remainder of this paper is organized as follows. Section 2 reviews the multirotor model and the geometric preliminaries on $\text{SO}(3)$. Section 3 states the tracking problem and defines the error variables. Section 4 presents the proposed finite-time position and attitude controller. Section 5 reports simulation results, and Section 6 concludes the paper.

2. PRELIMINARIES

This section introduces the dynamic model of the multirotor and the geometric tools on the special orthogonal group $\text{SO}(3)$. These elements serve as the mathematical foundation for the controller design presented in later sections.

The multirotor is modeled as a rigid body subject to positional and rotational dynamics. Let $x \in \mathbb{R}^3$ denote the position in the inertial frame and $v \in \mathbb{R}^3$ the linear velocity.

The positional motion is described by

$$\dot{x} = v, \quad (1)$$

$$\dot{v} = ge_3 + \frac{L}{m}, \quad (2)$$

where m is the vehicle mass, g is the gravitational constant, $e_3 = [0 \ 0 \ 1]^\top$, and $L \in \mathbb{R}^3$ is the control thrust expressed in the inertial frame.

The attitude of the multirotor is represented by a rotation matrix $R \in \text{SO}(3)$. Let $\omega \in \mathbb{R}^3$ be the body-frame angular velocity, and $\tau \in \mathbb{R}^3$ the control torque. The rotational dynamics satisfy

$$\dot{R} = R\hat{\omega}, \quad (3)$$

$$\dot{\omega} = J^{-1}[\tau - \omega \times (J\omega)], \quad (4)$$

where J is the inertia matrix and $\hat{\omega}$ is the skew-symmetric matrix

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}, \quad \hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \quad (5)$$

The hat and vee operators satisfy

$$(\hat{x})^\vee = x, \quad \forall x \in \mathbb{R}^3. \quad (6)$$

Because multirotor attitude evolves on the nonlinear Lie group $\text{SO}(3)$, appropriate geometric tools are required for analysis and controller construction.

The special orthogonal group is defined by

$$\text{SO}(3) = \{R \in \mathbb{R}^{3 \times 3} \mid R^\top R = I, \det(R) = 1\}. \quad (7)$$

Any vector $\phi \in \mathbb{R}^3$ corresponds to a rotation via the exponential map

$$\exp(\hat{\phi}) \in \text{SO}(3). \quad (8)$$

The logarithmic map provides a locally invertible mapping from $\text{SO}(3)$ to its Lie algebra $\mathfrak{so}(3)$. According to Johnson and Beard (2022), and Bullo and Lewis (2019), for a rotation matrix $R \in \text{SO}(3)$ with rotation angle

$$\theta = \arccos\left(\frac{\text{tr}(R) - 1}{2}\right), \quad (9)$$

the logarithmic map is given by

$$\text{LOG}(R) = \frac{\theta}{2 \sin \theta} (R - R^\top)^\vee. \quad (10)$$

This map will later be used to define attitude deviations in the tangent space of $\text{SO}(3)$, but no error construction is introduced at this stage.

3. PROBLEM FORMULATION

3.1 Reference Trajectories

Let $x_{ref}(t)$, $v_{ref}(t)$, $R_{ref}(t)$, $\omega_{ref}(t)$ denote the reference position, velocity, attitude, and angular velocity, respectively. All reference trajectories are assumed to be sufficiently smooth and bounded.

3.2 Finite-Time Tracking Objective and Modeling Scope

The control objective is to design the thrust L and torque τ such that the position, velocity, attitude, and angular-velocity tracking errors converge to zero in finite time. According to the fractional-exponent finite-time attractor introduced in (Yogi et al., 2021),

$$\dot{X} = -X - X^\alpha, \quad 0 < \alpha < 1, \quad (11)$$

this model captures the essential dynamic behavior required for finite-time convergence.

The coexistence of the linear and fractional-exponent terms induces an adaptive two-phase behavior: a smooth decay is governed by the linear component when the error is relatively large, while the nonlinear fractional term accelerates convergence as the state approaches the origin.

In both regions, the corresponding dominant term ensures that the error continues to converge rapidly toward zero.

The analysis in this study considers nominal multirotor dynamics and does not explicitly model external disturbances or parametric uncertainties. The formulation presented here therefore focuses exclusively on establishing finite-time tracking performance for the nominal system.

3.3 Problem Statement and Control Objective

The control objective is stated as follows:

- (1) **Given:** the nominal multirotor dynamics in Section 2 and smooth reference trajectories.
- (2) **Design:** the thrust vector L and control torque τ .
- (3) **Achieve:**
 - finite-time convergence of all position, velocity, attitude, and angular velocity tracking errors;
 - auxiliary variables satisfying a finite-time stable structure of the form (11) in both the translational and rotational subsystems.

4. FRACTIONAL EXPONENT SLIDING MODE CONTROLLER DESIGN

This section presents the finite-time sliding mode control framework developed for the multirotor. The controller is constructed in a cascaded manner, consisting of a position controller (outer-loop) and an attitude controller (inner-loop) on SO(3). Both subsystems follow the unified finite-time convergence principle introduced in Section 3.

4.1 Outer-Loop Control

The translational tracking errors are defined as

$$e_x = x - x_{ref}, \quad (12)$$

$$e_v = v - v_{ref}. \quad (13)$$

Their dynamics follow directly from the quadrotor model:

$$\dot{e}_x = e_v, \quad (14)$$

$$\dot{e}_v = ge_3 + \frac{L}{m} - \dot{v}_{ref}. \quad (15)$$

The electronic speed controllers (ESCs) are the primary hardware responsible for generating lift in multirotor UAVs. Together with the flight controller, they regulate the motor power and rotational speed, which directly determines the thrust. Since the ESC time constant is negligible compared to that of the rigid-body dynamics, the thrust magnitude is assumed to be instantaneously realized.

Under the assumption we have $f = f_{ref}$, where $f = \|L\|$, $f_{ref} = \|L_{ref}\|$. Consequently, only the thrust direction requires regulation. The inner-loop attitude controller is responsible for aligning the third body-axis with the reference counterpart generated by the outer loop. With this assumption, the thrust tracking error is expressed as

$$e_L = L - L_{ref} = -fb_3 + f_{ref}b_{3ref}, \quad (16)$$

where b_3 and b_{3ref} are the current and reference third body-axes, respectively. Since $f = f_{ref}$, therefore the expression for e_L simplifies to

$$e_L = -f_{ref}(b_3 - b_{3ref}). \quad (17)$$

Substitution of (16) into (15) yields

$$\dot{e}_v = ge_3 + \frac{1}{m}(L_{ref} + e_L) - \dot{v}_{ref}. \quad (18)$$

Finite-Time Sliding Surface Inspired by the fractional exponent finite-time attractor structure in (11), based on Yogi et al. (2021), the sliding surface is defined as

$$s_p = \beta_1 e_v + e_x + e_x^{\alpha_1}, \quad 0 < \alpha_1 < 1, \quad (19)$$

where $\beta_1 > 0$ is a design parameter. Its time derivative becomes

$$\dot{s}_p = \beta_1 \dot{e}_v + \dot{e}_x + \alpha_1 e_x^{\alpha_1-1} \odot \dot{e}_v, \quad (20)$$

where $\odot$ denotes the element-wise product. Substituting (18) into (20) yields

$$\dot{s}_p = \beta_1 \left[ge_3 + \frac{1}{m}(L_{ref} + e_L) - \dot{v}_{ref} \right] + e_v + \alpha_1 e_x^{\alpha_1-1} \odot e_v. \quad (21)$$

Finite-Time Convergence To facilitate control design, the thrust is decomposed as

$$L_{ref} = L_{ref1} + L_{ref2}. \quad (22)$$

Select L_{ref1} such that all terms unrelated to finite-time convergence are canceled:

$$L_{ref1} = m \left[\dot{v}_{ref} - \frac{1}{\beta_1} (e_v + \alpha_1 e_x^{\alpha_1-1} \odot e_v) - ge_3 \right]. \quad (23)$$

Thus,

$$\dot{s}_p = \frac{\beta_1}{m} (L_{ref2} + e_L). \quad (24)$$

To achieve finite-time convergence, the following control is chosen:

$$L_{ref2} = -\frac{m}{\beta_1} (s_p + s_p^{\epsilon_1}), \quad 0 < \epsilon_1 < 1. \quad (25)$$

Therefore, L_{ref} is given by

$$L_{ref} = m \left[\dot{v}_{ref} - \frac{1}{\beta_1} (e_v + \alpha_1 e_x^{\alpha_1-1} \odot e_v) - ge_3 \right] - \frac{m}{\beta_1} (s_p + s_p^{\epsilon_1}), \quad (26)$$

which leads to

$$\dot{s}_p = -s_p - s_p^{\epsilon_1} + \frac{\beta_1}{m} e_L. \quad (27)$$

When the inner-loop attitude controller drives the rotational error to zero, the thrust-direction error satisfies $e_L = 0$. In this case, the dynamics of the sliding variable reduces to the intended finite-time form

$$\dot{s}_p = -s_p - s_p^{\epsilon_1}, \quad (28)$$

ensuring finite-time convergence of s_p . It follows from standard finite-time stability results (Yogi et al., 2021) that the sliding variable satisfies

$$s_p(t) \rightarrow 0 \quad \text{in finite time.}$$

When the sliding condition $s_p = 0$ is reached, the algebraic relation

$$s_p = \beta_1 e_v + e_x + e_x^{\alpha_1} = 0,$$

implies

$$e_v = \dot{e}_x = -\frac{1}{\beta_1} (e_x + e_x^{\alpha_1}). \quad (29)$$

Equation (29) has the same finite-time attractor structure as (11); therefore $e_x(t)$ converges to zero in finite time.

Since e_v is an explicit nonlinear function of e_x through (29), the finite-time convergence of e_x directly implies finite-time convergence of e_v . Thus, the proposed outer-loop control guarantees that

$$e_x(t), e_v(t) \rightarrow 0 \quad \text{in finite time.}$$

Substituting the thrust command (26) into the error dynamics (18) yields the closed-loop velocity error dynamics

$$\dot{e}_v = -\frac{1}{\beta_1} (e_v + \alpha_1 e_x^{\alpha_1-1} \odot e_v + s_p + s_p^{\epsilon_1}) + \frac{1}{m} e_L, \quad (30)$$

where s_p was introduced earlier in (19).

4.2 Construction of the Reference Attitude R_{ref}

The reference attitude is determined by the thrust command L_{ref} . Define the reference body z -axis as :

$$b_{3ref} = -\frac{L_{ref}}{\|L_{ref}\|}. \quad (31)$$

The reference heading aligns with the projection of v_{ref} onto the plane orthogonal to b_{3ref} . Define

$$b_{1ref} = \frac{b_{3ref} \times (v_{ref} \times b_{3ref})}{\|v_{ref} \times b_{3ref}\|} \quad (32)$$

$$b_{2ref} = b_{3ref} \times b_{1ref} \quad (33)$$

Then the reference attitude matrix is

$$R_{ref} = [b_{1ref}, b_{2ref}, b_{3ref}]. \quad (34)$$

4.3 Inner-Loop Control

The attitude controller is developed within the geometric framework of $SO(3)$, ensuring that all tracking errors are treated intrinsically on the rotation manifold. Let $R \in SO(3)$ denote the current attitude and R_{ref} the reference attitude generated by the outer loop. Define

$$E_R = R^T R_{ref}. \quad (35)$$

The rotational tracking errors are

$$e_R = \text{LOG}(E_R) \quad (36)$$

$$e_\omega = E_R \omega_{ref} - \omega \quad (37)$$

Left Jacobian and Geometric Error Relation As recalled in Section 2.2.2, the Left Jacobian associated with a rotation vector $e_R = \theta u$ is

$$J_l = I + \frac{2}{\theta} \sin^2\left(\frac{\theta}{2}\right) \hat{u} + [1 - \text{sinc}(\theta)] \hat{u}^2, \quad (38)$$

$$J_l^{-1} = I - \frac{\theta}{2} \hat{u} + \left[1 - \frac{2}{\theta} \cot\left(\frac{\theta}{2}\right)\right] \hat{u}^2, \quad (39)$$

where $\theta = \|e_R\|$ and $u = e_R/\|e_R\|$, which is a unit vector.

Due to the curvature of $SO(3)$, the attitude error rate does not satisfy $\dot{e}_R = e_\omega$. The correct relation is

$$\dot{e}_R = J_l^{-1} e_\omega. \quad (40)$$

Differentiating (40) gives

$$\ddot{e}_R = \dot{J}_l^{-1} e_\omega + J_l^{-1} \dot{e}_\omega, \quad (41)$$

where the term $\dot{J}_l^{-1}$ is computable.

From equation (4) and (37), the angular velocity error dynamics are

$$\begin{aligned} \dot{e}_\omega &= E_R \dot{\omega}_{ref} - \hat{\omega} E_R \omega_{ref} - \dot{\omega} \\ &= E_R \dot{\omega}_{ref} - \hat{\omega} E_R \omega_{ref} + J^{-1} \hat{\omega} J \omega - J^{-1} \tau \\ &= A - J^{-1} \tau, \end{aligned} \quad (42)$$

where

$$A = E_R \dot{\omega}_{ref} - \hat{\omega} E_R \omega_{ref} + J^{-1} \hat{\omega} J \omega. \quad (43)$$

The notation above introduces A to represent all terms independent of the control input. This substitution is made to simplify subsequent expressions, since A appears repeatedly in the derivation of the sliding dynamics and the construction of the torque components.

Finite-Time Sliding Surface Motivated by the fractional exponent finite-time structure (11) based on Yogi et al. (2021), the attitude sliding surface is defined as

$$s_R = \beta_2 \dot{e}_R + e_R + e_R^{\alpha_2}, \quad (44)$$

where $\beta_2 > 0, 0 < \alpha_2 < 1$.

Differentiating (44) gives

$$\begin{aligned} \dot{s}_R &= \beta_2 \ddot{e}_R + J_l^{-1} e_\omega + \alpha_2 e_R^{\alpha_2-1} \odot J_l^{-1} e_\omega \\ &= \beta_2 \left[\dot{J}_l^{-1} e_\omega + J_l^{-1} \dot{e}_\omega \right] + B \end{aligned} \quad (45)$$

where

$$B = J_l^{-1} e_\omega + \alpha_2 e_R^{\alpha_2-1} \odot J_l^{-1} e_\omega. \quad (46)$$

Substitution of (42) into (45) yields

$$\begin{aligned} \dot{s}_R &= \beta_2 \left[\dot{J}_l^{-1} e_\omega + J_l^{-1} (A - J^{-1} \tau) \right] + B \\ &= \beta_2 \left(\dot{J}_l^{-1} e_\omega + J_l^{-1} A \right) + B - \beta_2 J_l^{-1} J^{-1} \tau \end{aligned} \quad (47)$$

Finite-Time Convergence To simplify the sliding dynamics, the torque input is decomposed as

$$\tau = \tau_1 + \tau_2. \quad (48)$$

The first component cancels all unwanted terms:

$$\tau_1 = \frac{1}{\beta_2} J J_l \left[\beta_2 \left(\dot{J}_l^{-1} e_\omega + J_l^{-1} A \right) + B \right]. \quad (49)$$

The second component attains finite-time error convergence:

$$\tau_2 = \frac{1}{\beta_2} J J_l (s_R + s_R^{\epsilon_2}), \quad 0 < \epsilon_2 < 1. \quad (50)$$

Thus, the sliding dynamics reduce to

$$\dot{s}_R = -s_R - s_R^{\epsilon_2}, \quad (51)$$

where τ is given by

$$\tau = J \left[A + \frac{1}{\beta_2} J_l \left(\beta_2 \dot{J}_l^{-1} e_\omega + B + s_R + s_R^{\epsilon_2} \right) \right] \quad (52)$$

It follows from standard finite-time stability results (Yogi et al., 2021) that the sliding variable satisfies

$$s_R(t) \rightarrow 0 \quad \text{in finite time.}$$

When the sliding condition $s_R = 0$ is reached, the algebraic relation

$$s_R = \beta_2 \dot{e}_R + e_R + e_R^{\alpha_2} = 0$$

implies

$$\dot{e}_R = J_l^{-1} e_\omega = -\frac{1}{\beta_2} (e_R + e_R^{\alpha_2}). \quad (53)$$

Since the inverse left Jacobian J_l^{-1} is bounded and non-singular for all e_R within the domain of interest, the multiplicative term J_l^{-1} does not affect the finite-time nature of the dynamics. Equation (53) has the same finite-time attractor structure as (11); therefore $e_R(t)$ converges to zero in finite time. Since e_ω is an explicit nonlinear function of e_R through (53), the finite-time convergence of e_R directly implies finite-time convergence of e_ω . Thus, the proposed inner-loop controller guarantees that

$$e_R(t), e_\omega(t) \rightarrow 0 \quad \text{in finite time.}$$

Substituting the torque command τ into the angular velocity error dynamics (42) yields the closed-loop dynamics

$$\dot{e}_\omega = -J_l J_l^{-1} e_\omega - \frac{1}{\beta_2} J_l (B + s_R + s_R^{\epsilon_2}) \quad (54)$$

where B and s_R were introduced in (46) and (44).

5. SIMULATION RESULTS

This section presents simulation results that validate the proposed linear-nonlinear sliding mode controller. All simulations are performed under the multirotor dynamics described in Section 2, using the parameters and controller gains listed in Table 1.

Table 1. Multirotor Parameters and Controller Gains

Parameter	Value
Mass m	2 kg
Inertia J	$\text{diag}(0.03, 0.03, 0.05) \text{ kg} \cdot \text{m}^2$
Outer-loop gains	$\beta_1 = 0.8, \alpha_1 = 0.6, \epsilon_1 = 0.6$
Inner-loop gains	$\beta_2 = 0.4, \alpha_2 = 0.6, \epsilon_2 = 0.6$

The evaluation is conducted using a smooth figure-eight trajectory, which is commonly employed to test multirotor agility, disturbance resilience, and nonlinear tracking performance. The reference trajectory is generated as

$$\mathbf{x}_{ref}(t) = \begin{bmatrix} 25 \sin(0.32t) \cos(0.32t) \\ 25 \sin(0.32t) \\ -10 - 1.5 \sin(0.1t) \end{bmatrix}.$$

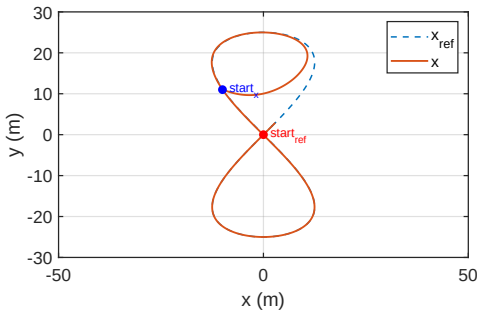


Fig. 1. Top-view of the actual trajectory and the reference counterpart.

The corresponding UAV flight path is compared with the reference in Figs. 1-2, demonstrating that the vehicle accurately follows the figure-eight motion.

The position tracking error is presented in Fig. 3, followed by the velocity tracking error in Fig. 4. Fig. 5 shows the thrust command generated by the outer-loop controller.

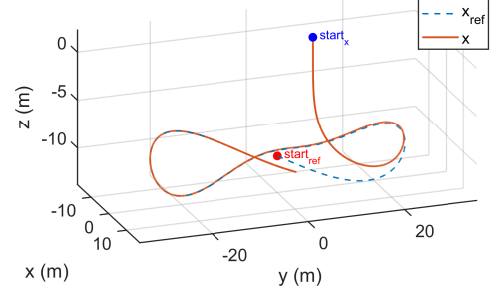


Fig. 2. 3D view of the actual trajectory and the reference counterpart.

Fig. 6 shows the attitude error defined using the logarithmic map on $\text{SO}(3)$, and Fig. 7 presents the angular velocity tracking error. Finally, Fig. 8 shows the torque command from the attitude controller.

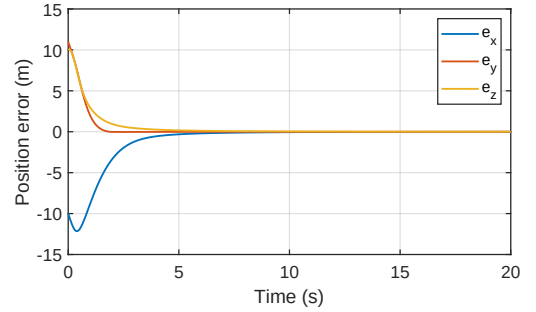


Fig. 3. Trajectory of position tracking error e_x .

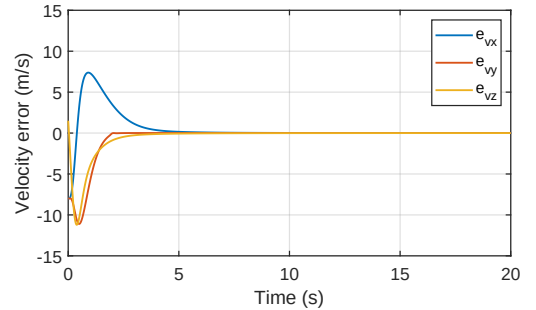


Fig. 4. Trajectory of velocity tracking error e_v .

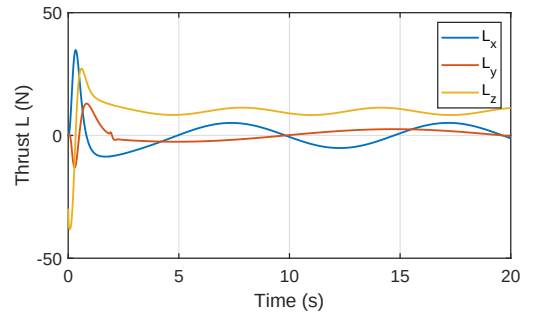


Fig. 5. Trajectory of thrust.

All position, velocity, attitude, and angular velocity errors converge to zero in finite time, confirming the effectiveness of the proposed control scheme.

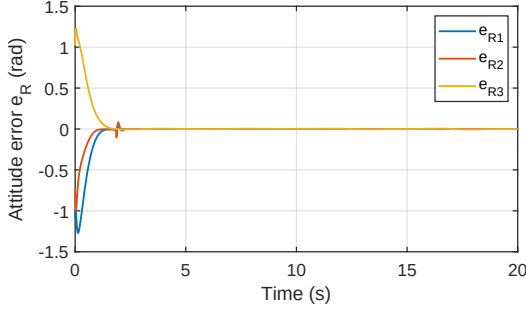


Fig. 6. Trajectory of attitude tracking error e_R .

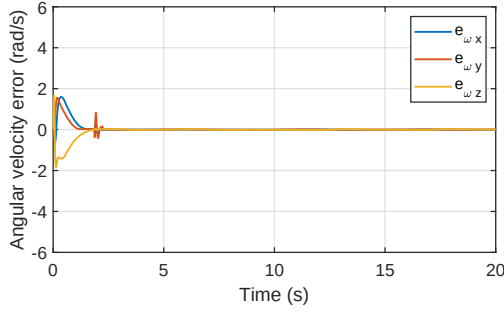


Fig. 7. Trajectory of angular velocity error e_ω .

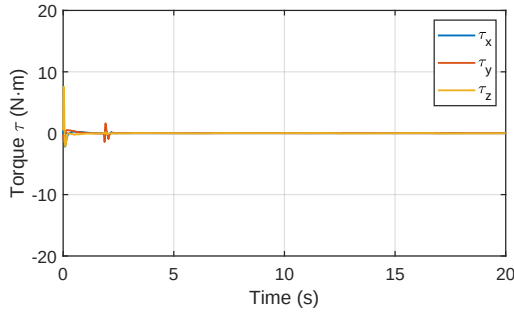


Fig. 8. Trajectory of torque.

6. CONCLUSION AND FUTURE WORK

This work proposes a unified linear–nonlinear sliding-mode framework for multirotor trajectory tracking. Both the position and attitude controllers adopt a finite-time error-convergence structure, with the attitude error defined through the logarithmic map on $SO(3)$. The sliding surfaces integrate linear terms with nonlinear fractional exponents to achieve fast convergence and improved transient behavior. Simulation results confirm the effectiveness and practical feasibility of the proposed approach.

Future work will incorporate disturbance observers or nonlinear estimation methods to better compensate for wind and model uncertainties in real-world flight.

DECLARATION OF GENERATIVE AI AND AI-ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work the author(s) used *ChatGPT* in order to assist with language refinement and grammar checking. After using this tool/service, the

author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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